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Optimization problem in Type II hybrid censoring with fixed and random sample size

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Abstract: In this paper, we study Type II hybrid censoring and the related problem of finding the optimal sample size. The Rayleigh distribution is considered for the lifetime distribution. Additionally, the sample size is assumed to be either a fixed value or a random variable drawn from a geometric distribution. One of the most important criteria for studying this problem is the cost associated with the experiment. Therefore, the optimal sample size is determined such that the cost function does not exceed a pre-defined value. In the case where the sample size is a random variable, the goal is to identify the optimal parameters for the distribution. To evaluate the obtained results, numerical computation, a simulation study, and a real example have been performed. Finally, the conclusions of the article are presented.

Keywords: Optimization problem; Random sample size; Type II hybrid censoring.
Mathematics Subject Classification (2010): 62N01, 62N05.

1 Introduction

In some lifetime tests, the experimenter may unable to record all the survival times of units under the experiment. For example, individuals in a clinical trial may drop out of the study, or the study may have to be terminated early for different reasons such as lack of time or funds. Also, in an industrial experiment, units may break accidently. Data obtained in these situations are called censored data. Censoring can be done in different ways. The most famous censoring methods are Type I and Type II censoring schemes. Hybrid censoring is a combination of Type I and Type II censoring schemes,

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which is divided into two types, Type I and Type II hybrid censoring schemes. In Type I hybrid censoring method which was first introduced by Epstein (1954), the test terminates in time $T = \min(X_{r:n}, \tau)$, when the values of τ and r are pre-determined values. This concept is known as Type I hybrid censoring scheme. The disadvantage of this method is that we might only have a very small number of failed units. As a result, a new censoring method known as Type II hybrid censoring was introduced by Childs et al. (2003). With this approach, the experiment ends at the time $T = \max(X_{r:n}, \tau)$. This scheme has the advantage of guaranteeing at least r failure times.

On the other hand, in some situations, it is not feasible to keep a constant sample size due to various reasons. In these cases we have a random number of units under the test. Srivastava (1973) provides several examples that illustrate these situations.

A critical practical challenge in designing a life test is determining the best sample size. Since larger samples and longer tests increase the total cost, a statistically robust test must also be economically viable. This leads to an optimization problem where a total cost function, typically incorporating costs related to sample size, failures, and test duration, is minimized subject to a maximum allowable budget. This problem was studied by many researchers. See, for example, Ebrahimi (1988), Pham (1992), Pham and Zhang (1999), Bhattacharya et al. (2014), Ahmadi et al. (2016), Cordeiro and Pham (2017), Basiri (2017), Bhattacharya et al. (2020), Basiri and Asgharzadeh (2021), Basiri and Hosseinzadeh (2022) and Basiri and MirMostafaee (2023).

The purpose of this article is to determine the best sample size in Type II hybrid censoring from the Rayleigh distribution, based on a cost criterion. A detailed overview of the paper is as below. In Section 2, we present the cost function which plays an essential role in this study. Then, assuming the sample size as a fixed value, the problem of finding the best sample size in Type II hybrid censoring is studied. Section 3 contains a similar problem when the sample size is a random variable. In this situation, the optimal parameter for the distribution of sample size, is the aim. A simulation study is provided in Section 4 for illustrating the theoretical results. In Section 5, one real data example is presented. Finally, the conclusion is expressed.

2 Description of model for fixed $N = n$

Throughout this paper, we assume that $X_1, \ldots, X_n$ is a sample of n units, from the Rayleigh distribution with the probability density function (pdf) and the cumulative distribution function (cdf) as

$$f_\alpha(x) = 2\alpha x e^{-\alpha x^2}, \quad \text{and} \quad F_\alpha(x) = 1 - e^{-\alpha x^2}, \quad x > 0, \alpha > 0. \tag{1}$$

Here, the corresponding order statistics are shown by $X_{1:n} \leq \cdots \leq X_{n:n}$ with the corresponding observed values as $x_{1:n} \leq \cdots \leq x_{n:n}$. Assuming Type II hybrid censoring, we denote D and $T = \max(X_{r:n}, \tau)$ as the number of failures and the duration of the test, respectively, where τ and r are the pre-fixed values. Clearly, D and T are both random variables. In the following, we develop the approach of determining the best value for the sample size. Towards this end, we first propose the following expected cost function which was used by Basiri and Hosseinzadeh (2022).

$$EC(n) = C_0 + C_n n + C_d E(D) + C_t E(T), \tag{2}$$

where C_0, C_n, C_d and C_t are the sampling set-up cost or any other related cost involved in sampling, the cost per unit, the cost per unit of failed item, and the cost per unit of duration of life-testing, respectively.

On the one hand, for Type II hybrid censoring there are three cases as

$$\begin{cases} \text{Case I}: & \{x_{1:n},\ldots,x_{r:n}\}, \ \text{if} \ \tau < x_{r:n}, \\ \text{Case II}: & \{x_{1:n},\ldots,x_{D:n}\}, \ \text{if} \ x_{r:n} < \cdots < x_{D:n} < \tau < x_{D+1:n}, \quad r \le D < n, \\ \text{Case III}: & \{x_{1:n},\ldots,x_{n:n}\}, \ \text{if} \ x_{n:n} < \tau. \end{cases}$$

Clearly, for Case I, $T = \max(x_{r:n}, \tau) = x_{r:n}$, so r failures took place. For Case II and Case III, $T = \tau$ which leads to j failures, $j = r, \ldots, n$. So, we have

$$\begin{aligned} P(D=r) &= \sum_{j=0}^{r} \binom{n}{j} (F_\alpha(\tau))^j \left(\bar{F}_\alpha(\tau)\right)^{n-j}, \\ P(D=j) &= \binom{n}{j} (F_\alpha(\tau))^j \left(\bar{F}_\alpha(\tau)\right)^{n-j}, \quad j = r+1, \ldots, n. \end{aligned}$$

Using (1) and the binomial expansion for $(F_\alpha(\tau))^j = \left(1 - \bar{F}_\alpha(\tau)\right)^j$ leads to

$$\begin{aligned} E(D) &= r\sum_{j=0}^{r} \binom{n}{j} (F_\alpha(\tau))^j \left(\bar{F}_\alpha(\tau)\right)^{n-j} + \sum_{j=r+1}^{n} j\binom{n}{j} (F_\alpha(\tau))^j \left(\bar{F}_\alpha(\tau)\right)^{n-j} \\ &= r\sum_{j=0}^{r}\sum_{k=0}^{j}\binom{n}{j}\binom{j}{k}(-1)^k(\bar{F}_\alpha(\tau))^{n-j+k} + \sum_{j=r+1}^{n}\sum_{k=0}^{j} j\binom{n}{j}\binom{j}{k}(-1)^k(\bar{F}_\alpha(\tau))^{n-j+k} \\ &= r\sum_{j=0}^{r}\sum_{k=0}^{j} B(j,k,n)e^{-(n-j+k)\alpha\tau^2} + \sum_{j=r+1}^{n}\sum_{k=0}^{j} jB(j,k,n)e^{-(n-j+k)\alpha\tau^2}, \end{aligned} \tag{3}$$

where

$$B(j,k,n) = \binom{n}{n-j,\, k,\, j-k}(-1)^k, \quad \text{for} \quad \binom{a}{b,c,d} = \frac{a!}{b!c!d!}, \quad \text{with} \quad a = b+c+d. \tag{4}$$

On the other hand, we can write

$$\begin{aligned} E(T) &= E(\max(X_{r:n}, \tau)) \\ &= \tau.P(X_{r:n} < \tau) + \int_\tau^\infty x f_{X_{r:n}}(x)dx \\ &= \tau.F_{X_{r:n}}(\tau) + \int_\tau^\infty x f_{X_{r:n}}(x)dx, \end{aligned}$$

where $f_{X_{r:n}}(\cdot)$ and $F_{X_{r:n}}(\cdot)$ are the pdf and cdf of $X_{r:n}$, respectively. Integrating by parts leads to

$$\begin{aligned} E(T) &= \tau.F_{X_{r:n}}(\tau) + \tau.\bar{F}_{X_{r:n}}(\tau) + \int_\tau^\infty \bar{F}_{X_{r:n}}(x)dx \\ &= \tau + \int_\tau^\infty \bar{F}_{X_{r:n}}(x)dx \end{aligned}$$

$$
\begin{aligned}
&= \tau + \sum_{j=0}^{r-1} \binom{n}{j} \int_{\tau}^{\infty} \left(F_\alpha(x)\right)^j \left(\bar{F}_\alpha(x)\right)^{n-j} dx \\
&= \tau + \sum_{j=0}^{r-1} \sum_{k=0}^{j} B(j,k,n) \int_{\tau}^{\infty} e^{-(n-j+k)\alpha x^2} dx,
\end{aligned}
$$

where the last equality is obtained by using the binomial expansion for $(F_\alpha(x))^j = \left(1 - \bar{F}_\alpha(x)\right)^j$ and Equations (1) and (4). Now, we can write

$$
\begin{aligned}
\int_{\tau}^{\infty} e^{-(n-j+k)\alpha x^2} dx &= \sqrt{\frac{\pi}{(n-j+k)\alpha}} \int_{\sqrt{2(n-j+k)\alpha\tau}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz \\
&= \sqrt{\frac{\pi}{(n-j+k)\alpha}} \left[1 - \Phi\left(\sqrt{2(n-j+k)\alpha\tau}\right)\right],
\end{aligned}
$$

in which $z = \sqrt{2(n-j+k)\alpha} x$ and $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. This leads to

$$
E(T) = \tau + \sum_{j=0}^{r-1} \sum_{k=0}^{j} B(j,k,n) \sqrt{\frac{\pi}{(n-j+k)\alpha}} \left[1 - \Phi\left(\sqrt{2(n-j+k)\alpha\tau}\right)\right]. \tag{5}
$$

Table 1 reports the values of $EC(n)$ based on Equations (2), (3) and (5) for different choices of n, r and τ, when $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$. From Table 1, by an empirical evidence, we get the values of $EC(n)$ increase along with the values of r, n and τ increase, when all other parameters are kept fixed. In fact, it was expected because increasing the values of r, n and τ means that we can have more failed items. Also, by increasing the value of τ it is observed that the values of $EC(n)$ are quite close to the each others by changing the values of r. This suggests that when the maximum time τ is high, it tends to dominate the stopping rule, reducing the influence of r on the total expected cost.

Table 1: The values of $EC(n)$ when $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$.

n	$r/\ \tau$	1	2	5	7
10	2	39.3221	44.8168	51.0000	55.0000
	5	39.4810	44.8168	51.0000	55.0000
	7	40.3044	44.8169	51.0000	55.0000
20	5	75.6425	84.6337	91.0000	95.0000
	10	75.7595	84.6337	91.0000	95.0000
	15	78.1297	84.6337	91.0000	95.0000
50	10	184.6063	204.0842	211.0000	215.0000
	20	184.6067	204.0842	211.0000	215.0000
	30	185.3121	204.0842	211.0000	215.0000
	40	193.0050	204.0842	211.0000	215.0000

For the special case that $\tau = 0$, we have the Type II censoring. As $EC(n)$ is an increasing function of τ, the values of $EC(n)$ for the Type II censoring are smaller than other cases.

One can see that the expected cost function defined in (2) is a nonlinear function in decision variable n. Here, we derive the best value for n, say n_{opt}, such that the

Table 2: The values of n_{opt} with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$.

c^*	$r/\ \tau$	1	2	5	7
50	1	$\{1,\dots,12\}$	$\{1,\dots,11\}$	$\{1,\dots,9\}$	$\{1,\dots,8\}$
	3	$\{3,\dots,12\}$	$\{3,\dots,11\}$	$\{3,\dots,9\}$	$\{3,\dots,8\}$
	5	$\{5,\dots,12\}$	$\{5,\dots,11\}$	$\{5,\dots,9\}$	$\{5,\dots,8\}$
	7	$\{7,\dots,12\}$	$\{7,\dots,11\}$	$\{7,8,9\}$	$\{7,8\}$
	10	$\{10,11,12\}$	$\{10,11\}$	$-$	$-$
	15	$-$	$-$	$-$	$-$
	20	$-$	$-$	$-$	$-$
	30	$-$	$-$	$-$	$-$
	40	$-$	$-$	$-$	$-$
100	1	$\{1,\dots,26\}$	$\{1,\dots,23\}$	$\{1,\dots,22\}$	$\{1,\dots,21\}$
	3	$\{3,\dots,26\}$	$\{3,\dots,23\}$	$\{3,\dots,22\}$	$\{3,\dots,21\}$
	5	$\{5,\dots,26\}$	$\{5,\dots,23\}$	$\{5,\dots,22\}$	$\{5,\dots,21\}$
	7	$\{7,\dots,26\}$	$\{7,\dots,23\}$	$\{7,\dots,22\}$	$\{7,\dots,21\}$
	10	$\{10,\dots,26\}$	$\{10,\dots,23\}$	$\{10,\dots,22\}$	$\{10,\dots,21\}$
	15	$\{15,\dots,26\}$	$\{15,\dots,23\}$	$\{15,\dots,22\}$	$\{15,\dots,21\}$
	20	$\{20,\dots,26\}$	$\{20,\dots,23\}$	$\{20,21,22\}$	$\{20,21\}$
	30	$-$	$-$	$-$	$-$
	40	$-$	$-$	$-$	$-$

expected cost function is less than a pre-fixed value for the budget, say c^*. This difficult optimization problem cannot be solved analytically and numerical computations should be utilized. Table 2 shows the values of n_{opt} for different choices of r and τ, with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 50,\ 100$. From Table 2 we get that for the large values of r there is no n_{opt}, which satisfies the condition $EC(n) \leq c^*$. These cases are indicated by a dash $(-)$ in Table 2. For the other cases, the smallest value for n_{opt} is r and the largest one decreases as τ increases. From Table 2, we find that n_{opt} is not unique. Figure 1 shows the values of $EC(n)$ with respect to n for different choices of r and τ with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 50,\ 100$. Figure 1 confirms the results in Table 2.

3 Description of model for random N

Let $X_{1:n} \leq \cdots \leq X_{N:N}$ be the order statistics associated with a sample of size N from the Rayleigh distribution. Independently, let N be a non-negative integer-valued random variable whose domain is $\{r, r+1, \dots\}$. We assume that N follows the geometric distribution truncated at point r, say $Ge(1-p; r)$, with probability mass function (pmf) as

$$P(N = n) = (1-p)p^{n-r}, \qquad n = r, r+1, \dots,\ r \geq 1, \quad 0 \leq p \leq 1. \tag{6}$$

From (6), it is easy to show that $E_N(N) = r + \frac{p}{1-p}$, which is increasing in p. For finding the best values for the sample size we should find the best value for p. To this end, we consider the modified expected cost function as

$$EC(p) = C_0 + C_n E_N(N) + C_d E_N\left(E(D|N = n)\right) + C_t E_N\left(E(T|N = n)\right).$$

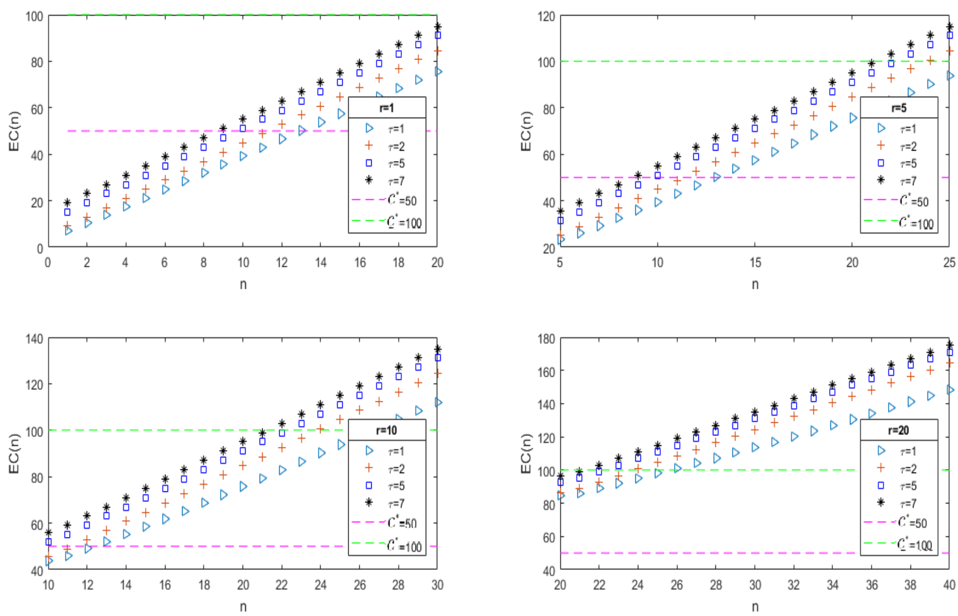

Figure 1: The plots of $EC(n)$ with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$.

The optimal values for p are ones that the condition $EC(p) \leq c^*$ is satisfied, where c^* is a pre-fixed value. Also, we have

$$
\begin{aligned}
E_N\left(E(D|N=n)\right) &= \sum_{n=r}^{\infty} E(D|N=n)(1-p)p^{n-r}, \\
E_N\left(E(T|N=n)\right) &= \sum_{n=r}^{\infty} E(T|N=n)(1-p)p^{n-r},
\end{aligned}
$$

where the values of $E(D|N=n)$ and $E(T|N=n)$ are defined in (3) and (5), respectively.

In Table 3, the values of the expected cost function $EC(p)$ for different choices for τ, r and p are reported, when $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$. From Table 3 one can find that the values of $EC(p)$ are increasing in τ, r and p, when all other parameters are held fixed.

In the following, it is tried to find the optimal value for p, say p_{opt}, in such a way that $EC(p) \leq c^*$, where c^* is a pre-fixed value. Table 4 presents the values of p_{opt} for different choices of r and τ, with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 50,\ 100$. From Table 4 we observe that, when p_{opt} exists, the upper values are decreasing in r and τ. Figure 2 confirm the results of Table 4.

Table 3: The values of $EC(p)$ when $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 0$ and $C_t = 2$.

p	$r/\ \tau$	1	2	5	7	10
0.1	1	7.452	9.513	15.517	19.517	25.517
	5	23.704	25.796	31.807	35.807	41.807
	10	44.057	46.149	52.167	56.167	62.167
	15	64.416	66.500	72.527	76.527	82.527
	20	84.773	86.851	92.884	96.884	102.884
0.3	1	8.535	10.757	16.771	20.771	26.771
	5	24.617	26.962	32.996	36.996	42.996
	10	44.858	47.218	53.277	57.277	63.277
	15	65.130	67.475	73.557	77.557	83.557
	20	85.408	87.732	93.836	97.836	103.836
0.5	1	10.530	13.013	19.040	23.040	29.040
	5	26.378	29.139	35.202	39.202	45.202
	10	46.452	49.297	55.403	59.403	65.403
	15	66.617	69.457	75.603	79.603	85.603
	20	86.807	89.618	95.803	99.803	105.803
0.7	1	15.280	18.302	24.358	28.358	34.358
	5	30.781	34.349	40.454	44.454	50.454
	10	50.537	54.408	60.575	64.575	70.575
	15	70.494	74.469	80.696	84.696	90.696
	20	90.542	94.531	100.816	104.816	110.816
0.9	1	39.265	44.638	50.800	54.789	60.773
	5	54.145	60.506	66.723	70.706	76.681
	10	73.003	80.266	86.543	90.515	96.473
	15	92.033	99.889	106.211	110.163	116.091
	20	111.110	119.278	125.620	129.539	135.418

Table 4: The values of p_{opt} with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$.

c^*	$r/\ \tau$	1	2	5	7	10
50	1	[0,1]	[0,1]	[0,0.8960],	[0,0.8758]	[0,0.8455]
	3	[0,1]	[0,0.8869]	[0,0.8555]	[0,0.8353]	[0,0.8048]
	5	[0,0.8765]	[0,0.8463]	[0,0.8149]	[0,7837]	[0,0.6862]
	7	[0,0.8329]	[0,0.8055]	[0,0.7227]	[0,6236]	[0,0.1098]
	10	[0,0.6793]	[0,0.5371]	–	–	–
	15	–	–	–	–	–
	20	–	–	–	–	–
	30	–	–	–	–	–
	40	–	–	–	–	–
100	1	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
	3	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
	5	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
	7	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
	10	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
	15	[0,1]	[0,1]	[0,0.8673]	[0,0.8463]	[0,0.8147]
	20	[0,0.8284]	[0,0.7852]	[0,0.6743]	[0,0.5107]	–
	30	–	–	–	–	–
	40	–	–	–	–	–

4 Simulation study

In this section, we run a simulation study to assess the performances of the results in Sections 2 and 3. The following algorithm has been applied for this purpose:
(i) Let the values of r, τ, α, c^*, C_0, C_n, C_d and C_t be known;

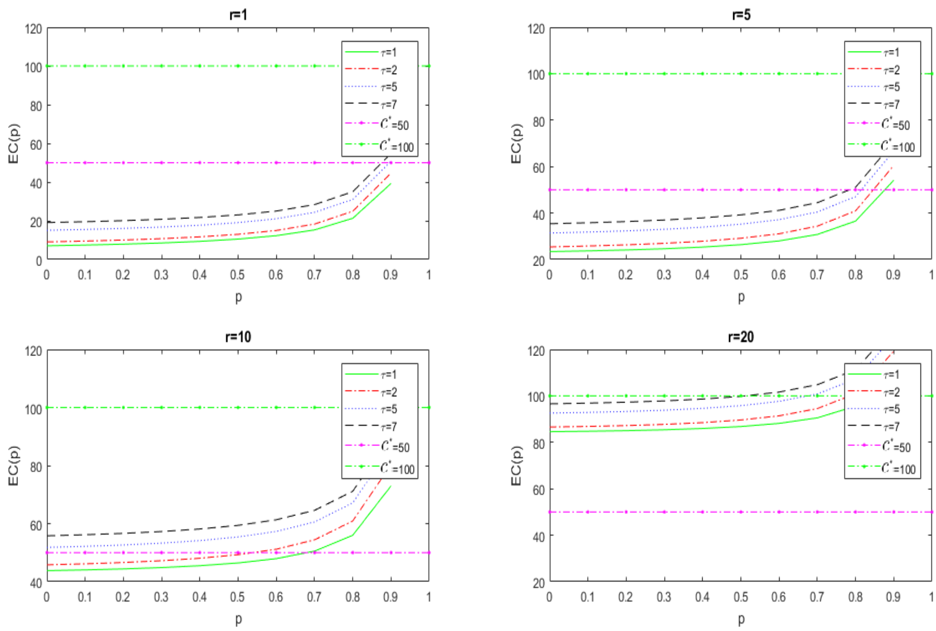

Figure 2: The plots of $EC(p)$ with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$ and $C_t = 2$.

(ii) Select n_{opt} (p_{opt}) such that the condition $EC(n) \leq c^*$ ($EC(p) \leq c^*$) is satisfied.
(iii) For the random case, generate N from the distribution $Ge(1-p_{opt}; r)$, say $n_{opt} = N$.
(iv) Generate n_{opt} iid random variables $X_1, \ldots, X_{n_{opt}}$ from the Rayleigh distribution with parameter α. Then sort them as $X_{1:n_{opt}} \leq \cdots \leq X_{n_{opt}:n_{opt}}$.
(v) Set $T^* = \tau$ if $\tau > X_{r:n_{opt}}$, else set $T^* = X_{r:n_{opt}}$.
(vi) Set $D^* = \sum_{j=1}^{n_{opt}} \mathbb{I}(X_{j:n_{opt}} \leq \tau)$ if $\tau > X_{r+1:n_{opt}}$, else set $D^* = r$, where $\mathbb{I}(\cdot)$ denotes the indicator function.
(vii) Repeat Steps (ii)-(vi) for $K = 10^4$ times and let $n_{opt}(i)$, $D^*(i)$ and $T^*(i)$ be the associated results in the i-th iteration, $i = 1, \ldots, K$. Then, calculate the average cost (AC) function by using

$$AC = C_0 + \frac{C_n}{K}\sum_{i=1}^{K} n_{opt}(i) + \frac{C_d}{K}\sum_{i=1}^{K} D^*(i) + \frac{C_t}{K}\sum_{i=1}^{K} T^*(i).$$

Based on the algorithm mentioned above, we have computed the values of AC with different choices of r and τ for $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 50$. All the obtained results are reported in Table 5. For Table 5 we have selected the largest values of n_{opt} and p_{opt} from Tables 2 and 4, respectively. We mention that the results reported in Table 5 are based on 10000 Monte Carlo simulations. As expected, from Table 5 it is observed that in all cases $AC \leq c^*$. It should be noted that all the computations in this paper have been done using MATLAB software.

Table 5: The values of (n_{opt}, AC) and (p_{opt}, AC) with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 50$.

	(n_{opt}, AC)		(p_{opt}, AC)	
$r/\ \tau$	1	2	1	2
1	(12, 46.499)	(11, 48.798)	(1, 40.260)	(1, 40.458)
3	(12, 46.580)	(11, 48.799)	(1, 40.275)	(0.8869, 41.874)
5	(12, 46.628)	(11, 48.799)	(0.8765, 40.451)	(0.8463, 47.896)
7	(12, 46.650)	(11, 48.801)	(0.8329, 41.152)	(0.8055, 42.056)
10	(12, 47.109)	(11, 48.809)	(0.6793, 43.356)	(0.5371, 43.416)

5 Real data analysis

Here we consider $n = 23$ deep-groove ball bearing failure times. This data set originally used by Lieblein and Zelen (1956), is also discussed in Caroni (2002). The observations are the number of million revolutions before failure for each 23 ball bearings ordered according to life endurance. The adequacy of the fitness of the one parameter Rayleigh distribution with $\alpha = \frac{n}{\sum_{i=1}^{n} x_i^2} = 1.52$ to the data is tested using the Kolmogorov-Smirnov (K-S) test. The value of K-S test statistic is obtained as $D = 0.13739$ with a corresponding $p - value = 0.7781$. Hence, the one parameter Rayleigh distribution fits the data quite well. For having the corresponding values from the one parameter Rayleigh distribution with $\alpha = 1$ we multiply the values by $\sqrt{1.52}$. Here, we consider the values $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ with the budget constraint $c^* = 100$. Based on the result in Table 2, $n_{opt} = 23$ is the optimal value for $\tau = 1, 2$ and $r = 1, \ldots, 20$. So, we present the results in Table 6. From Table 6 it can be seen that for all cases, the condition $EC(n) \leq c^*$ is held.

Table 6: The values of $EC(n_{opt})$ for $n_{opt} = 23$ and different choices of r and τ with $\alpha = 1$, $C_0 = 1$, $C_n = 3$, $C_d = 1$, $C_t = 2$ and $c^* = 100$.

		$\tau = 1$			$\tau = 2$		
r	$x_{r:n_{opt}}$	T	D	$EC(n_{opt})$	T	D	$EC(n_{opt})$
1	0.2207	1	15	87	2	22	96
3	0.4074	1	15	87	2	22	96
5	0.5200	1	15	87	2	22	96
7	0.5985	1	15	87	2	22	96
10	0.6681	1	15	87	2	22	96
15	0.8503	1	15	87	2	22	96
20	1.3067	1.3067	20	92.6	2	22	96

6 Conclusion

In this paper, the best sample size in Type II hybrid censoring from the Rayleigh distribution is derived. Two cases as fixed and random sample sizes are considered in this paper. For each case, a cost function is introduced and the best sample size is determined in such a way that the cost function does not exceed a pre-fixed value. For the random case, since the distribution of the sample size depends on the parameter p, the optimal value for p is the aim. Although in this paper we have mainly addressed the best sample size, one can consider the determination of the optimal value for (n, r, τ) (or (p, r, τ)) simultaneously using any criterion. This would be taken up in the future study.

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