Journal of Statistical Modelling: Theory and Applications
Vol. 2, No. 1, 2021, pp. 59-65
Yazd University Press 2021

JSMTA

Research Paper

Importance of components in a consecutive-(k, k)-out-of-n:F system having two dependent subcomponents based on Birnbaum measure

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Received: July 12, 2021/ Revised: November 18, 2021 / Accepted: November 18, 2021

Abstract: Measuring the total contribution of the components in a system is useful for various goals. Therefore, several component importance measures have been introduced in the literature. One of the well-known measures is the Birnbaum component importance measure. This paper concentrates on obtaining the Birnbaum component importance of the components of a consecutive-(k, k)-out-of-n:F system having two dependent subcomponents. A numerical example is also presented to illustrate the obtained results.

Keywords: Birnbaum component importance, Consecutive-(k, k)-out-of-n:F system, Dependence.
Mathematics Subject Classification (2010): 62N05, 60K10.

1 Introduction

The total contribution of the components plays an important role in system reliability. The importance measures rank the components by evaluating the system reliability in terms of the component reliability. It helps the engineers to evaluate the reliability of systems at various stages such as design, improvement, and control. In reliability literature, various measures of importance have been introduced by Barlow and Proschan (1975), Birnbaum (1969), Boland and El-Neweihi (1995), Eryilmaz (2017), Kuo and Zhu (2012), Wu and Coolen (2013) and Zhu and Kuo (2014).

Birnbaum (1969) introduced the importance of the i-th component in a coherent system. Consider a binary coherent system with n independent components. Suppose

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that X_i denotes i-th component state which $X_i=1$ if the component is working and $X_i=0$ if the component has failed and the event S shows that the system works. The Birnbaum reliability importance of the i-th component is defined by

$$I_i = P(S|X_i=1) - P(S|X_i=0).$$

In the literature, the reliability analysis of coherent systems that consist of n single (independent/dependent) components has been extensively studied and their properties and characteristics are discussed through various theoretical and methodological approaches. Although in these studied systems, the components are single elements, in some practical applications, engineers encounter systems consisting of components with two or more subcomponents. Cha and Finkelstein (2016) introduced (r,s)-out-of-n systems and studied the reliability and mean residual life functions of some complex systems consisting of n elements, each element having two s-dependent components. Eryilmaz (2017) introduced systems having two dependent subcomponents and investigated its reliability and properties for a consecutive-(k,k)-out-of-n:F system. In this context, we study the reliability importance for his proposed system.

Consider a consecutive-(k,k)-out-of-n:F system that includes n independent components each having two dependent subcomponents $(A_i,B_i), i=1,\ldots,n$. The system is assumed to function if and only if both systems of subcomponents $A_1,\ldots,A_n$ and $B_1,\ldots,B_n$ work under certain structural rules. The bivariate vector $(x_i,y_i), i=1,\ldots,n$ denotes the state of i-th component where x_i is the state of the first subcomponent A_i and y_i is the state of the second subcomponent B_i. In this system, the structure function is defined as

$$\phi(x_1,y_1,\ldots,x_n,y_n) = \begin{cases} 1, & \prod_{j=1}^{n-k+1}\left(1-\prod_{i=j}^{j+k-1}(1-x_i)\right)=1 \; and \\ & \prod_{j=1}^{n-k+1}\left(1-\prod_{i=j}^{j+k-1}(1-y_i)\right)=1, \\ 0, & o.w. \end{cases}$$

The bivariate state vectors (X_i,Y_i) for all $i=1,\ldots,n$ are independent and identical with the probabilities $p_{00}=P(X_i=0,Y_i=0)$, $p_{10}=P(X_i=1,Y_i=0)$, $p_{01}=P(X_i=0,Y_i=1)$, $p_{11}=P(X_i=1,Y_i=1)$, where $p_{00}+p_{10}+p_{01}+p_{11}=1$. Note that the i-th component fails if both of its subcomponents A_i and B_i fail, $i=1,\ldots,n$. The dependence of the subcomponents is involved in the reliability evaluation through the joint probabilities p_{00}, p_{10}, p_{01}, and p_{11}. Then, the reliability of this system is given as follows

$$R=\sum_{k_1=1}^{n}\sum_{k_2=1}^{n}\sum_{a=a_L}^{a_U} r_n(k_1,k_2,a)p_{00}^{a}p_{10}^{n-a-k_2}p_{01}^{n-a-k_1}p_{11}^{k_1+k_2+a-n}, \tag{1}$$

where $a_L=\max(0,n-k_1-k_2)$, $a_U=\min(n-k_1,n-k_2)$, $k_1(k_2)$ is the number of working components among $A_1,\ldots,A_n(B_1,\ldots,B_n)$, a denotes the number of failed components and $r_n(k_1,k_2,a)$ represents the number of path sets of the system including k_1 working subcomponents from $A_1,\ldots,A_n$, k_2 working subcomponents from $B_1,\ldots,B_n$, and total of a failed components.

The rest of the paper is organized as follows. In Section 2, for a consecutive-(k,k)-out-of-n:F system, the reliability importance of component is obtained based on the probabilities of functioning subcomponents. The illustrative example is presented to evaluate the reliability importance for different components of this system and ordinary (k,k)-out-of-n:F system in Section 3 and the comparison results are investigated. Concluding remarks are given in Section 4.

2 The Birnbaum reliability importance

In this section, the Birnbaum reliability importance is discussed based on the states of subcomponents. Consider a consecutive-(k,k)-out-of-n:F system with independent components each having two dependent subcomponents. Suppose that the event S shows that the system works and $C_i = 1-(1-X_i)(1-Y_i)$ denotes the state of i-th component which is 1 if this component works and is 0 if this component fails. The Birnbaum reliability importance of components i can be represented as

$$\begin{aligned}
I_i &= P(S|C_i=1) - P(S|C_i=0)\\
&= P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|((0,1)\cup(1,0)\cup(1,1)))\\
&\quad -P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(0,0))\\
&= \frac{1}{p_{01}+p_{10}+p_{11}}\Big[p_{01}P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(0,1))\\
&\quad +p_{10}P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(1,0))+p_{11}P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(1,1))\Big]\\
&\quad -P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(0,0)),
\end{aligned} \tag{2}$$

where $(i',j')=(X_i=i',Y_j=j')$, $i',j'=0,1$ and by considering $E=(0,1)$, $F=(1,0)$ and $G=(1,1)$, the third equality is obtianed as follows

$$\begin{aligned}
P(S|(E\cup F\cup G)) &= \frac{P(S\cap(E\cup F\cup G))}{P(E\cup F\cup G)}\\
&= \frac{P((S\cap E)\cup(S\cap F)\cup(S\cap G))}{P(E\cup F\cup G)}\\
&= \frac{P(S\cap E)+P(S\cap F)+P(S\cap G)}{P(E)+P(F)+P(G)} \quad (E,F \text{ and } G \text{ are disjoint})\\
&= \frac{P(E)P(S|E)+P(F)P(S|F)+P(G)P(S|G)}{P(E)+P(F)+P(G)}\\
&= \frac{p_{01}P(S|E)+p_{10}P(S|F)+p_{11}P(S|G)}{p_{01}+p_{10}+p_{11}}.
\end{aligned}$$

According to definition of conditional probability and using Equation (1), the probabilities are obtained as follows

$$P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(1,1)) = \frac{P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1,(1,1))}{P(X_i=1,Y_i=1)}$$

$$= \sum_{k_1=2}^{n}\sum_{k_2=2}^{n}\sum_{a=a_L}^{a_U} r_n^{11}(k_1,k_2,a)p_{00}^{a}p_{10}^{n-a-k_2}p_{01}^{n-a-k_1}p_{11}^{k_1+k_2+a-n-1},$$

$$P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(1,0))$$
$$= \sum_{k_1=2}^{n}\sum_{k_2=1}^{n-1}\sum_{a=a_L}^{a_U} r_n^{10}(k_1,k_2,a)p_{00}^{a}p_{10}^{n-a-k_2-1}p_{01}^{n-a-k_1}p_{11}^{k_1+k_2+a-n},$$

$$P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(0,1))$$
$$= \sum_{k_1=1}^{n-1}\sum_{k_2=2}^{n}\sum_{a=a_L}^{a_U} r_n^{01}(k_1,k_2,a)p_{00}^{a}p_{10}^{n-a-k_2}p_{01}^{n-a-k_1-1}p_{11}^{k_1+k_2+a-n},$$

$$P(\phi(X_1,Y_1,\ldots,X_n,Y_n)=1|(0,0))$$
$$= \sum_{k_1=1}^{n-1}\sum_{k_2=1}^{n-1}\sum_{a=a_L}^{a_U} r_n^{00}(k_1,k_2,a)p_{00}^{a-1}p_{10}^{n-a-k_2}p_{01}^{n-a-k_1}p_{11}^{k_1+k_2+a-n},$$

where $a_L=\max(0,n-k_1-k_2), a_U=\min(n-k_1,n-k_2)$, k_1,k_2 and a are mentioned for Equation (1) and also $r_n^{11}(k_1,k_2,a), r_n^{10}(k_1,k_2,a), r_n^{01}(k_1,k_2,a)$ and $r_n^{00}(k_1,k_2,a)$ are the number of path sets of the system including k_1 working subcomponents from A_i, k_2 working subcomponents from B_i, and a total of a failed components which respectively symbolize the situation of both two subcomponents work, of first subcomponent works and second fails, of first subcomponent fails and second works and both two subcomponents fail.

3 Illustrative example

In this section, illustrative results are presented to observe and evaluate the values of Birnbaum reliability importance for a consecutive- and ordinary-(k,k)-out-of-n:F system for different components.

Example 3.1. *Consider a (k,k)-out-of-n:F system with p_{00},p_{10},p_{01} and p_{11} be the probabilities of subcomponents state as*

p_{00}	p_{10}	p_{01}
0.05	0.10	0.20
0.20	0.15	0.15
0.10	0.07	0.63
0.01	0.08	0.09

Here, the component importance is calculated for consecutive- and ordinary-$(2,2)$-out-of-3:F system. The number of paths set for different components with respect to all possible values of k_1,k_2 and a is given in Table 1. For these two systems, the values of Birnbaum's reliability importance are calculated in Table 2. It is observed that the values of Birnbaum reliability importance for the second component is larger in the consecutive structure, that is, the second component is more effective in system reliability than the first and third components. But in the ordinary structure, the converse is true, that is, the first and third components are more effective in system reliability than the second *component, except when the probability of functioning the subcomponent A_i is extremely more than the probability of functioning the subcomponent B_i.*

Table 1: The number of paths set for different components.

(1, 1)				
			$r_n^{11}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
2	2	1	2	2
2	2	0	2	2
2	3	0	2	2
3	2	0	2	2
3	3	0	1	1

(1, 0)				
			$r_n^{10}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
2	1	0	1	0
2	1	1	1	0
2	2	0	2	2
3	1	0	1	0
3	2	0	1	1

(0, 0)				
			$r_n^{00}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
1	1	2	1	0
1	2	1	1	0
2	1	1	1	0
2	2	1	1	1

(0, 1)				
			$r_n^{01}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
1	2	0	1	0
1	2	1	1	0
1	3	0	1	0
2	2	0	2	2
2	3	0	1	1

(1, 1)				
			$r_n^{11}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
1	1	2	1	0
1	2	1	2	0
1	2	0	0	0
1	3	0	1	0
2	1	1	2	0
2	1	0	0	0
2	2	1	2	2
2	2	0	2	2
2	3	0	2	2
3	1	0	1	0
3	2	0	2	2
3	3	0	1	1

(1, 0)				
			$r_n^{10}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
1	2	0	1	0
2	2	0	2	2
3	2	0	1	1

(0, 1)				
			$r_n^{01}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
2	1	0	1	0
2	2	0	2	2
2	3	0	1	1

(0, 0)				
			$r_n^{00}(k_1,k_2,a)$	
k_1	k_2	a	Consecutive	Ordinary
2	2	1	1	1

4 Conclusions

In this paper, the reliability importance in a consecutive k-out-of-n:F system having two dependent subcomponents was studied. The system is assumed to function if and only if both systems of subcomponents $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ work under certain structural rules. The reliability importance was obtained for this consecutive-(k, k)-out-of-n system. The effect of the contribution of one component is established in reliability of the system. Illustrative numerical results represented that the values of Birnbaum reliability importance is sensitive to the components position and the values

Table 2: Birnbaum reliability importance for consecutive- and ordinary-$(2,2)$-out-of-3:F system.

				I_i	
p_{00}	p_{10}	p_{01}	i	Consecutive	Ordinary
0.05	0.1	0.2	1	0.217	0.393
			2	0.449	0.229
			3	0.217	0.393
0.2	0.15	0.15	1	0.221	0.397
			2	0.533	0.355
			3	0.221	0.397
0.1	0.63	0.07	1	0.109	0.130
			2	0.312	0.240
			3	0.109	0.130
0.01	0.09	0.08	1	0.147	0.280
			2	0.297	0.180
			3	0.147	0.280

of probabilities of components state. Also in the consecutive structure, the position and probabilities of components are more significant than the ordinary structure. Generalization of the results of this paper based on some newer importance measure is the subject of our future work and we hope that to publish the new results soon.

Acknowledgement

We would like to thank and express our deep gratitude to referee for his useful comments, which led to the improved version of this manuscript.

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