A Critique on Power Spectrum – Area Fractal Method for Geochemical Anomaly Mapping

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(Received: March 2018, Accepted:October 2019)

Keywords

Two-Dimensional Fourier Transformation

Frequency Domain of Geochemical Data

Principal Component Analysis (PCA)

Power Spectrum – Area Fractal Method

Abstract

Power spectrum – area fractal (S-A fractal) method has been frequently applied for geochemical anomaly mapping. Some researchers have performed this method for separation of geochemical anomaly, background and noise and have delineated their distribution maps. In this research, surface geochemical data of Zafarghand Cu-Mo mineralization area have been utilized and some defects of S-A fractal method have been discussed. The surface geochemical data were transformed to the frequency domain using Fourier transformation and the S-A fractal method was performed on obtained Cu power spectrum. 4 geochemical classes were distinguished on the basis of fractal diagram then these classes were separated using various filters and their signals were analyzed separately by principal component analysis (PCA) and the situation of mineralization was interpreted. PCA shows the low frequency geochemical signals have strongly been affected by the Cu and Mo mineralization process. In the end, the Cu geochemical anomaly map based on this low frequency class was delineated using inverse Fourier transformation. The deep borehole that was drilled in the center of this obtained anomaly shows there is a mineralization zone at the depth. The disadvantages of S-A fractal method have been discussed using these obtained results.

1. INTRODUCTION

Interpretation of geochemical data is an important subject in the mineral explorations. Fractal analysis is a conventional approach for geochemical mapping. The variety of geology units, alterations and mineralization processes can cause the various geochemical societies. Fractal methods can distinguish and discriminate these geochemical classes. Until now, various fractal methods such as concentration-area, concentration-volume have been performed for interpretation of geochemical data [1-4]. The S-A fractal method was proposed for determination of geochemical societies in the frequency domain. This method separates the geochemical patterns on the basis of the feature of self-similarity in geochemical signals [5]. The geochemical data have more been interpreted in the spatial domain. In addition to the spatial domain, the frequency domain can also be used for accessing the exploratory information in the geochemical data. The frequency spectrum analysis of geochemical elements in the frequency domain has been used to identify the behavioral patterns of the elements [6 - 10].

The new exploratory information can be obtained using the frequency domain of geochemical data that cannot be easily obtained from the spatial domain. Using the frequency domain, it can be shown that there is a relationship between the surface frequency signals and the depth of the mineral deposits [10]. Some researchers have used the S-A fractal method in the frequency domain [4, 5, 11, 12, 13]. The S-A fractal method has also been applied on the mineralization factors derived from the PCA method [14].

In the S-A fractal method, the different frequency groups of the data are analyzed. This method assumes that very high frequencies are caused by the geochemical noise and the intermediate frequencies are related to the mineralization and geochemical anomalies and the low frequencies are related to the geochemical background of elements. This paper shows that this assumption has flaw and the very low frequencies include the strong mineralization patterns thus should not be filtered as the background component. PCA has been used for discussing of this issue.

Proper exploratory information can be obtained using PCA as a dimension reduction method. Determining the elements associated with mineralization and identifying the mineralization patterns are important issues in mineral exploration. In order to detecting the mineralization factor and determining the paragenesis elements, PCA method has been performed on the spatial and frequency domain of geochemical data. The PCA as a multivariate analysis method is a useful tool for combining several dependent variables and reducing the dimension of the data set in the independent principal components based on the covariance and correlation coefficients [15].

2. THE POWER SPECTRUM-AREA FRACTAL METHOD

A: Transferring the geochemical data from the spatial domain to the frequency domain: After interpolating the geochemical data and mapping the raster data, the geochemical distribution map is transformed to the frequency domain using two-dimensional Fourier transformation. The Fourier transformation of a function is calculated as follows [16, 17]:

$$\mathrm{Ff}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega x} f(x)dx \tag{1}$$

$Ff(\omega)$ is the Fourier transformation of f(x) function. The different signals in the geochemical distribution map can be discriminated using the Fourier transformation. The Fourier transformation algorithm is implemented in the MATLAB software.

B: Delineating the logarithmic graph of fractal: The values of the logarithmic power spectrum values versus the logarithm of cumulative corresponding areas are plotted. Using this graph, we can fit several straight lines to the data and determine the breakdown points of the graph, which show the threshold values.

The S-A fractal method consists of high-frequency spectral energy densities S [A (> S)] in a two-dimensional frequency domain and the fractal relationship is as follows:

$$A(\geq S) \alpha S^{-2d/\beta} \tag{2}$$

Where β represents the amount of anisotropy and d is the generalized scale invariance parameter (GSI) [18]. For a two-dimensional linear case, the relation is defined as follows:

$$A(\geq S) \alpha S^{-2/\beta} \tag{3}$$

C: Data filtering and inverse Fourier transformation: At this stage, the filters are designed based on the threshold values and applied to the data. The results are then transferred to the spatial domain using the inverse Fourier transformation and different maps are obtained accordingly [19]. The following equation shows the inverse Fourier transformation [16, 17].

$$f(x) = \frac{\mathbf{1}}{\mathbf{2\pi}} \int_{-\infty}^{+\infty} \mathrm{Ff}(\omega) e^{i\omega x} d\omega \tag{4}$$

3. GEOLOGY, MINERALIZATION AND ALTERATIONS OF THE AREA

Zafarghand area has been located in the northeast of Isfahan and 22 km the south of Ardestan city and 6 km the west of Zafarghand village. This area has been located on the Uromia-Dokhtar volcanic belt and at the edge of the central Iran zone, so it may have significant potential for copper or other elements (Figure 1) [20].

Lithological units of this exploration area mainly consist of dacite, andesite, basalt, diorite and alluvium. Most of these rock units have strongly been altered by the hydrothermal solutions and have created alteration zones from the center of the porphyry system to the margins including phyllic, argillic and propylitic. The potassic alteration is also observed in some parts of the region. Iron hydroxides (hematite, goethite and jarosite) are most commonly seen in relation to the central alterations. The siliceous veins and the stockwork veinlets of quartz-magnetite that have filled the joints are related to the copper mineralization (malachite and azurite). The phyllic and potassic alterations, such as iron oxide alteration, are strongly observed with the quartz diorite and porphyry dacite at the center of the alteration system. The most important rocks in this porphyry system are dacite porphyry, rhyolite dacite porphyry and quartz diorite surrounded by the porphyry andesite and andesitic pyroclasts. In some parts, the quartz veins are observed with the copper mineralization up to 4 meters thick and 30 meters long. Chalcopyrite, pyrite, galena, sphalerite, malachite and iron oxide minerals are present in the silicified and quartz veins. The potassic alteration is found in the microdiorites in the southern region of Zafarghand. The propylitic alteration is widespread throughout the region and surrounds the argillic alterations in the north, south, and west of area [20].

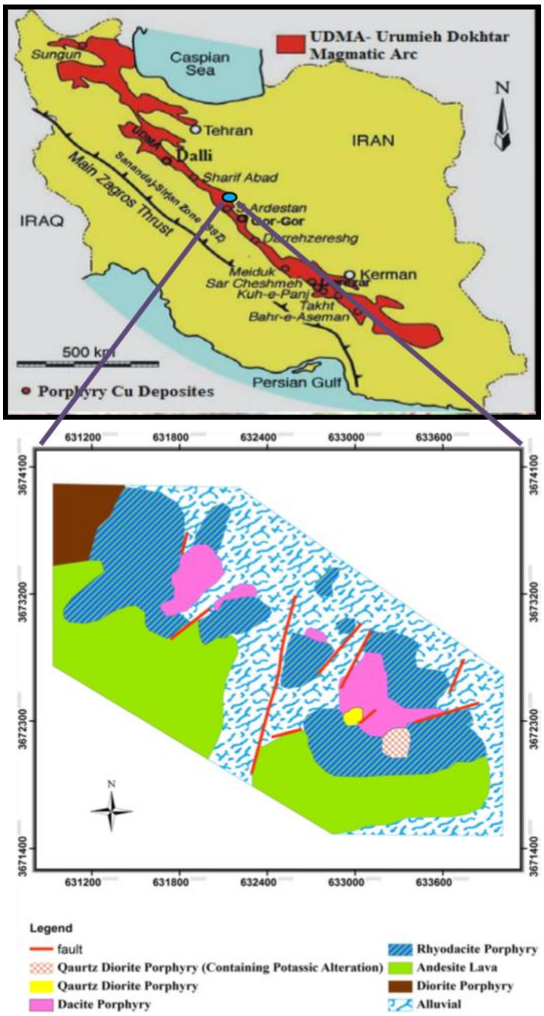

Figure 1. Geological map and location of the Zafarghand area in the Uromia - Dokhtar belt [10].

4. RESULTS AND DISCUSSION

In this area, 177 geochemical samples were taken and analyzed by ICP method for 43 elements. The location map of these samples is shown in figure 2.

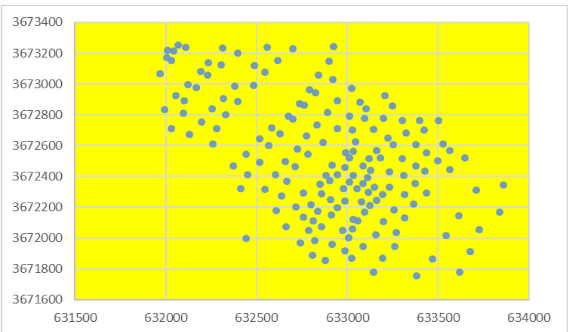

Figure 2. The location of geochemical samples in the study area

Identification of frequency characteristics of elements is important for achieving the exploratory information and investigating the situation of mineralization. In this study, the S-A fractal method and the PCA method were used together for detecting the geochemical properties of mineralization elements. This combined method can show the difference of frequency behavior of mineralization elements with background elements. Firstly, the geochemical data for all of elements were transformed to the frequency domain using 2 dimensional Fourier transformation. The power spectrum values of elements and the wave numbers in the horizontal and vertical direction were obtained using this transformation.

Figure 3 shows the distribution of logarithmic values of the Cu power spectrum values for different wavelengths in the Zafarghand area. The horizontal and vertical axes show the values of the wave number in the x and y directions respectively.

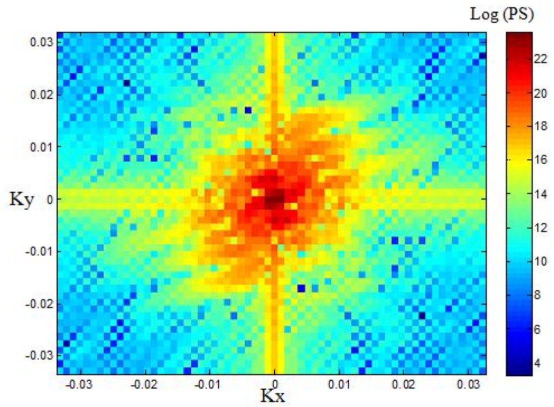

Figure 3. The distribution map of the Cu power spectrum values in the study area

The power spectrum values are calculated for each wave number and plotted logarithmically. As the power spectrum values increase, the frequency and wave numbers decrease.

The elements that are related to each other and hold common genesis will be similar in their power distribution map, especially if they have similar mobility powers. For this reason, the paragenesis elements have similar frequency behaviors and hence interesting information about the mineralization processes can be obtained in the frequency domain.

After transferring data to the frequency domain, the fractal method is applied on the power spectrum values of the mineralization element. According to the fractal method, the power spectrum values of the copper element were divided into 4 classes (Figure 4). A summary of the attributes of these classes are seen in Table 1. The Cu frequency data were divided into 4 classes with low, low to medium, medium to high, and high frequencies on the basis of the fractal method. Class 1 shows the values of the low power spectrum values that are related to the high frequencies. In the S-A fractal method, this class is known as geochemical noise, and these signals are filtered and removed from the data. Classes 2 and 3 are related to the intermediate power spectrum values and their frequencies are considered as anomalous signals in the S-A fractal method. Class 4 is related to the high power spectrum values and shows the low frequency geochemical signals. In the S-A fractal method, these signals are assumed as the geochemical background and they are removed from the data using a low-pass filter. According to the S-A fractal method only class 2 and 3 are related to the mineralization process and there are no mineralization effects in the class 1 and especially in the class 4. While this is not accurate and sometimes the high and low frequency signals include the important information about the mineralization processes and should not be completely removed from the data. The strong effects of mineralization on the low and high frequencies of geochemical data have also been previously detected [10, 21, 22]. In order to investigating this important issue, the frequency classes obtained from the S-A fractal method have been interpreted in this study.

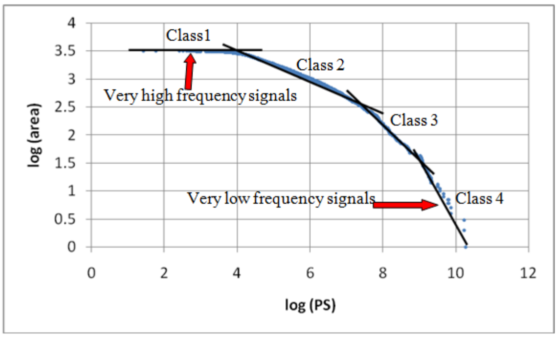

Figure 4. The S-A fractal diagram for the copper element in the frequency domain

Table 1. The characteristics of frequency classes obtained based on the S-A fractal method for Cu element

Frequency classes	Low threshold for power spectrum	High threshold for power spectrum	Frequency signals	Interpretation based on the S-A fractal method
Class1	26	10000	High	noise
Class2	10000	3194550	Moderate to high	Anomaly
Class3	3194550	364863612	Low to moderate	Anomaly
Class4	364863612	18310000000	low	background

The frequency signals in the class 4 were studied to investigate the mineralization attributes at the very low frequencies and to evaluate the relationship between these frequencies and the mineralization process. Therefore, the frequencies of this class were separated from the other frequencies using the low pass filter and eliminate the rest of the frequencies. The frequency distribution map of class 4 has been plotted in figure 5. As can be seen, these values are at the center of the power spectrum distribution map and, on the other hand, the magnitude of the frequency spectrum of this class is larger than the other classes that indicates the high amplitude of these signals in this frequency class. The frequency signals with high amplitude hold important role in reconstructing the initial two-dimensional wave, which is the geochemical distribution map of the element.

Therefore, the class 4 with high power spectrum values plays an important role in the reconstruction of the Cu geochemical distribution map in the region. The significance of this frequency class was investigated by PCA.

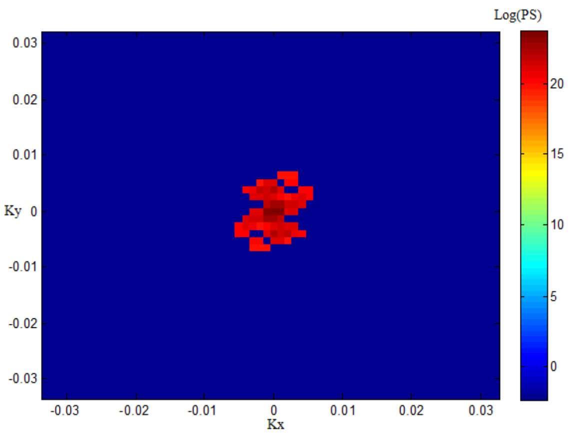

Figure 5. The Cu power spectrum distribution map in the class 4 using low pass filter based on the S-A fractal method

The various frequency classes were separated using different filters based on the obtained threshold values.

In order to determine the mineralization status in different frequency classes and to study the behavior of elements related to the mineralization process, the PCA method was performed on these classes separately. The results of PCA for classes 2, 3, and 4 are shown in Table 2. This table shows the principal components extracted from the data for each class based on the correlation coefficients of the elements. The paragenesis elements that are related to the mineralization process hold higher values in the mineralization principal component (table 2).

The principal components related to the mineralization and their elements have been identified in the each of these classes. The values greater than 0.6 have been designated as significant values in this table. In class 2, which corresponds to moderate to high frequencies, component 3 includes Ag, Mo, S and Pb and has been known as mineralization factor. As can be seen, copper is not found in any of the factors and has no specific role in this frequency class. This demonstrates the low importance of the Cu element in this frequency class, while Cu is the most important mineralization element in the region.

Table 2. The results of PCA on the class 2, 3 and 4 based on the S-A fractal method

	Components in class4			Components in class3					Components in class2				
	1	2		1	2	3	4		1	2	3	4	5
Au	0.95	0.30	Au	0.77	0.24	0.21	0.32	Au	0.44	0.42	0.43	0.61	-0.13
Al	0.97	0.26	Al	0.16	0.82	0.25	0.17	Al	0.33	0.79	0.37	0.27	-0.04
Ca	0.95	0.30	Ca	0.18	0.93	0.14	0.10	Ca	0.45	0.33	0.02	0.78	0.11
Fe	0.96	0.27	Fe	0.27	0.86	0.16	0.19	Fe	0.14	0.71	0.25	0.39	0.40
K	0.94	0.33	K	0.92	0.28	0.22	-0.04	K	0.84	0.31	0.13	0.14	0.27
Mg	0.95	0.32	Mg	0.30	0.95	0.01	0.00	Mg	0.29	0.33	0.18	0.86	0.09
Na	0.95	0.31	Na	0.96	0.11	0.17	0.12	Na	0.42	0.77	0.45	0.06	0.00
Ag	0.96	0.28	Ag	0.27	0.14	0.65	0.39	Ag	0.18	0.44	0.76	0.33	0.10
As	0.91	0.40	As	0.27	0.14	0.84	-0.02	As	0.81	-0.06	0.55	0.10	0.05
Ba	0.95	0.31	Ba	0.88	0.44	-0.03	-0.03	Ba	0.24	0.49	0.25	0.69	0.18
Be	0.96	0.26	Be	0.45	0.77	0.28	-0.08	Be	0.88	0.29	0.29	0.10	0.01
Bi	0.97	0.26	Bi	0.67	0.62	0.13	0.10	Bi	0.06	0.38	0.32	0.69	0.33
Cd	0.95	0.30	Cd	0.94	0.21	0.13	0.08	Cd	0.21	0.68	0.28	0.16	0.38
Ce	0.97	0.26	Ce	0.82	0.35	0.31	0.06	Ce	0.90	0.35	0.02	0.06	0.11
Co	0.95	0.31	Co	0.30	0.94	0.05	0.09	Co	0.08	0.46	0.18	0.78	0.33
Cr	0.62	0.74	Cr	0.18	0.84	0.24	-0.06	Cr	0.66	0.64	0.20	0.19	0.14
Cs	0.97	0.26	Cs	0.42	0.30	0.58	0.07	Cs	0.74	0.21	0.59	0.13	0.07
Cu	0.33	0.91	Cu	0.03	0.10	0.49	0.66	Cu	0.34	0.20	-0.02	0.05	0.47
La	0.97	0.26	La	0.83	0.18	0.40	0.07	La	0.95	0.13	0.11	0.16	0.09
Li	0.93	0.35	Li	0.21	0.92	0.10	0.17	Li	0.10	0.74	0.24	0.57	0.11
Mn	0.93	0.38	Mn	0.73	0.54	0.18	0.20	Mn	0.22	0.78	0.37	0.24	0.24
Mo	-0.12	0.96	Mo	0.25	0.38	0.53	0.34	Mo	0.14	0.30	0.89	0.18	0.14
Nb	0.96	0.27	Nb	0.21	0.93	0.15	0.05	Nb	0.71	0.06	0.45	0.52	-0.02
Ni	0.81	0.53	Ni	0.15	0.93	0.09	0.10	Ni	0.13	0.83	0.28	0.41	0.08
P	0.96	0.28	P	0.53	0.78	-0.02	0.24	P	0.13	0.25	0.78	0.52	0.04
Pb	0.63	0.59	Pb	0.38	0.25	0.51	0.35	Pb	0.16	0.08	0.95	0.11	0.08
Rb	0.96	0.29	Rb	0.88	0.20	0.35	-0.09	Rb	0.94	0.19	0.11	0.10	0.17
S	0.51	0.63	S	0.50	0.72	0.14	0.14	S	0.16	0.45	0.62	0.46	0.23
Sb	0.97	0.26	Sb	0.62	0.29	0.42	0.12	Sb	0.19	0.18	0.85	0.12	0.28
Sc	0.96	0.29	Sc	0.38	0.89	0.16	-0.07	Sc	0.68	0.57	0.20	0.35	0.09
Sn	0.97	0.26	Sn	0.21	0.21	0.10	0.84	Sn	0.28	0.12	0.26	0.18	0.69
Sr	0.95	0.31	Sr	0.66	0.60	0.14	0.14	Sr	0.13	0.93	0.12	0.22	0.11
Te	0.97	0.26	Te	0.88	0.22	0.11	0.22	Te	0.53	0.49	0.63	0.11	0.05
Th	0.96	0.27	Th	0.93	0.33	0.07	0.02	Th	0.66	0.08	0.22	0.61	0.29
Ti	0.96	0.27	Ti	0.50	0.75	0.32	0.13	Ti	0.87	0.16	0.24	0.37	0.08
Tl	0.97	0.26	Tl	0.72	0.19	0.52	0.10	Tl	0.92	-0.08	0.31	0.09	0.13
U	0.97	0.26	U	0.88	0.43	0.01	0.14	U	0.17	0.94	0.24	0.08	-0.05
V	0.95	0.31	V	0.35	0.91	0.16	-0.01	V	0.70	0.36	0.07	0.54	0.21
W	0.96	0.29	W	0.83	0.49	0.15	-0.03	W	0.15	0.22	0.93	0.09	0.13
Y	0.95	0.32	Y	0.73	0.47	0.33	0.11	Y	0.53	0.28	0.79	0.08	-0.01
Yb	0.96	0.29	Yb	0.70	0.59	0.22	0.11	Yb	0.46	0.33	0.81	0.08	0.01
Zn	0.95	0.31	Zn	0.78	0.05	0.35	0.42	Zn	0.07	0.09	0.25	0.43	0.71
Zr	0.96	0.28	Zr	0.14	0.89	0.12	0.30	Zr	0.14	0.91	0.07	0.31	0.09

In class 3, which includes low to moderate frequencies, the principal component 4 has been considered as a mineralization factor and contains only Cu element. In this frequency class, Mo element does not play an important role in any of the principal factors and the frequency behavior of this element is not significant in this class. Since the mineralization in this area is Cu-Mo type, none of the frequency classes in the mid-frequency range do not illustrate these two elements as paragenesis elements in the mineralization process. The behavior of these two elements in the class 2 and 3 are not similar together and are not very different from the background elements.

In other words, the mineralization factor in these frequency classes is not very prominent and this reduces the importance of these frequency classes in determining the mineralization characteristics. The results of the PCA on the class 4 which consists of very low frequencies are also shown in the table 2 and significant results have been achieved in this frequency class.

The PCA as a dimension reduction method can reduce the dimension of the geochemical variables based on the similarity and correlation between the elements. The PCA extremely reduced 43 features of the class 4 to 2 factors (table 2). This reduction in the number of features is a significant consequence and rarely occurs in data analysis field. This interesting result is directly related to the specific relations between the features (elements) in this dataset.

The Cu and Mo elements are together in the mineralization factor. The Cr and S elements are also found in this factor. The other elements are in the first factor which is the geochemical background factor. In this frequency class, the mineralization elements including copper and molybdenum are properly separated from other elements. The other elements that are considered as the background elements are in factor 1. These large numbers of elements behave quite similar to each other and behave differently to the mineralization elements in the frequency domain. The results in the class 4 show a very strong distinction between the mineralization and background elements, and copper and molybdenum are also well associated.

These results of frequency class 4 are more appropriate than the class 2 and 3 that related to the moderate frequencies and are more consistent with the mineralization reality of the region. The background elements in the class 4 have more similarity behaviors than the moderate frequency classes. Figure 6 illustrates how copper and molybdenum elements are separated from the other elements in the class 4 using the PCA method.

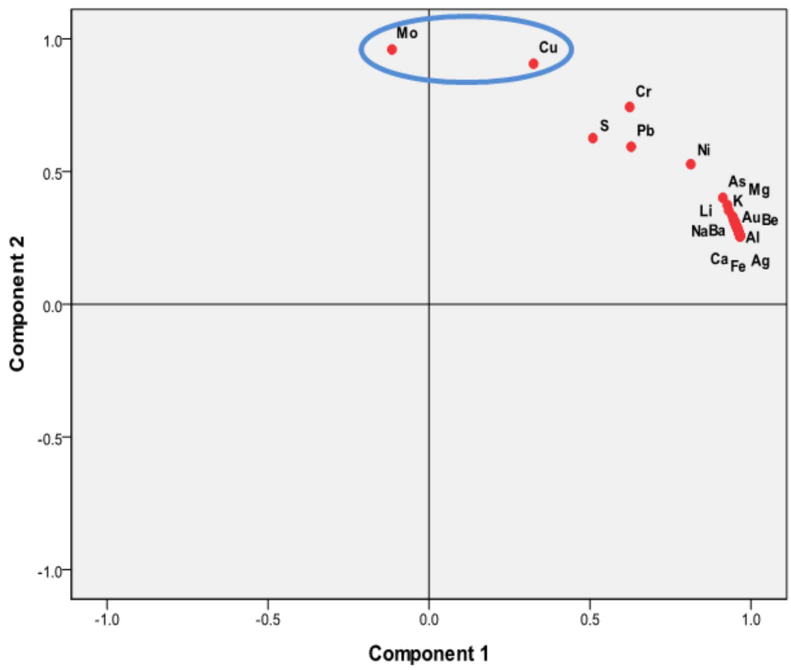

Figure 6. Separation of the mineralization elements from the background elements using the principal components obtained by PCA on the frequency class 4.

The remarkable results obtained in the class 4 show there are very strong effects of mineralization in these frequency data. Hence, this frequency class contains important exploratory information that can be used for identifying the mineralization elements and geochemical anomaly mapping. While, according to the S-A fractal method this frequency class is supposed as geochemical background and filtered from the data. Filtering and removing the frequency signals of this class will cause the losing a lot of exploratory information. These frequency signals that contain the significant information can be used for determination of the anomaly area. Figure 7 shows the Cu geochemical distribution map that obtained using the geochemical data in spatial domain and consists the all of frequency signals.

In order to determine the anomaly areas of the class 4, this frequency class was separated from the other frequencies and was transformed to the spatial domain using the inverse Fourier transformation method. The resulting map shows the distribution of copper values in the spatial domain corresponding to the low frequencies (Figure 8). The deep drilled borehole in the area are located precisely on the anomalies of this map, and good results have been obtained.

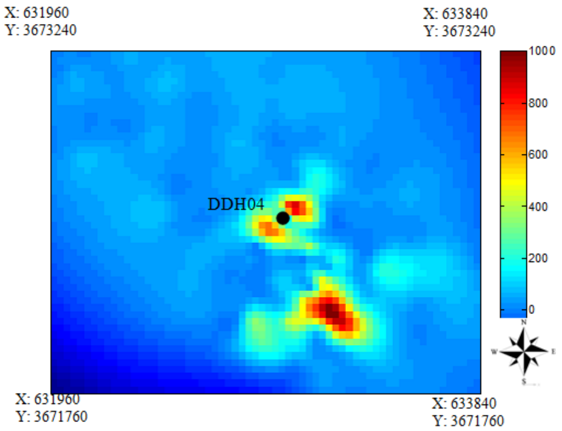

Figure 7. The Cu geochemical anomaly map derived from the spatial geochemical data (containing all frequencies in the data)

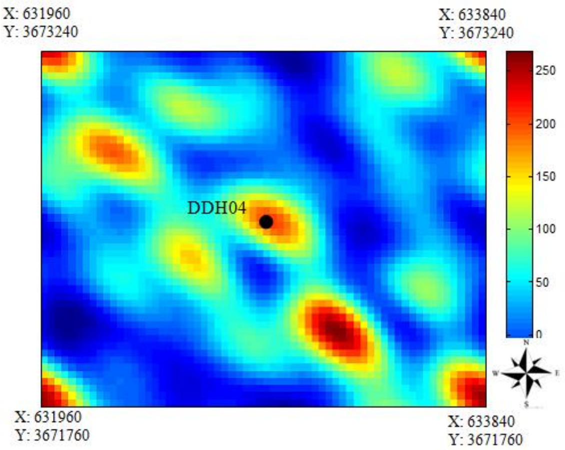

Figure 8. The Cu geochemical anomaly map obtained by class 4 and inverse Fourier transformation in the fractal method (low frequencies in Cu geochemical data)

The distribution of S, Cu and Mo in different depths of this borehole indicates there is a mineralization zone at the depth (Figures 9 and 10). The mineralization elements of deep ore deposits may migrate with low intensity to the surface and therefore be distributed weakly over the surface and produce the low frequency signals. Thus the mineralization elements can be distinguishable from the background elements in the low frequency signals. The location of these deep mineralizations can be detected using the low frequency geochemical signals.

There is an interesting relationship between the nature of the frequencies of surface geochemical data and the depth of mineralization [10]. Deep mineralization generates very low frequencies at the surface. The all of low frequencies in the geochemical data are not related to the background and should not be removed from the data.

Based on the conventional process in the S-A fractal method, the low frequency signals as background values are removed from the data. The results of this study demonstrates that these frequencies provide very useful exploratory information about the mineralization and by omitting these frequencies we lose a lot of information. The results showed that the frequency class 4 with very low frequencies hold much more information about the mineralization than the intermediate frequency classes.

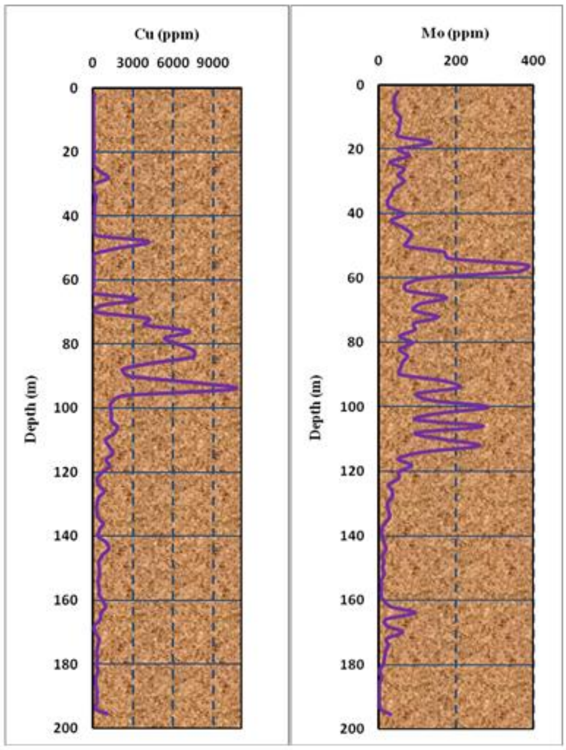

Figure 9. The variations of Cu and Mo concentrations at different depths and presence of the deep mineralization zone in the drilled borehole

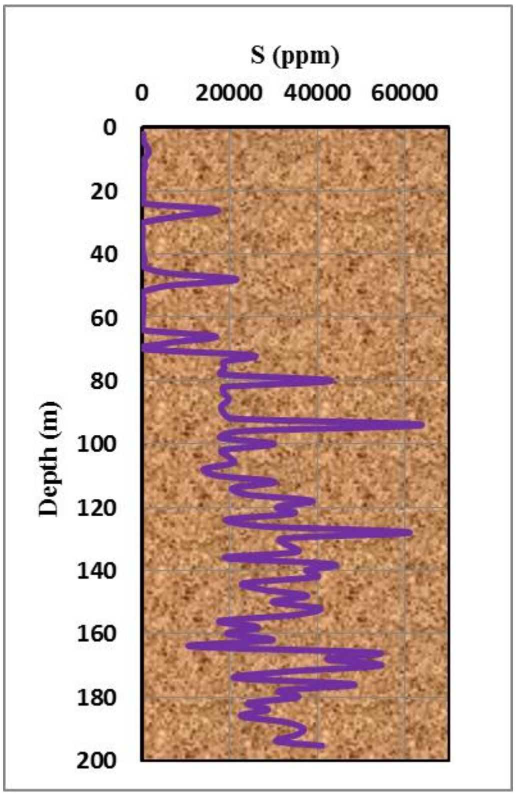

Figure 10. The variations of sulfur element at different depths in the drilled borehole

5. CONCLUSIONS

In this paper, the validity and disadvantages of the S-A fractal method was investigated in a case study of Zafarqhand copper-molybdenum mineralization zone. After transferring the geochemical data of 43 elements to the frequency domain, the S-A fractal method was applied on the Cu power spectrum values and four frequency classes were obtained accordingly. To investigate the effects of mineralization process in these four frequency classes, the PCA was performed on these classes separately. Finally, the geochemical anomaly map of the class 4 containing low frequencies was obtained by the inverse Fourier transformation. The following results were obtained on the basis of this study:

- The PCA as a dimension reduction tool notably reduced the number of 43 features (elements) into 2 factors in the class 4. The high intensity of the dimension reduction in the low frequency class indicates the existence of important information in this class.

- There are strong effects of mineralization in the low frequency signals in the class 4 which was properly detected using the PCA method. In this frequency class, mineralization elements including the copper and molybdenum were completely distinguished from other elements.

- The anomaly distribution map was obtained for class 4 using the two-dimensional inverse Fourier transformation. The results of the drilled borehole show a good coincidence with the anomaly map obtained from the Class 4. This study indicates the importance of the low frequency signals of geochemical data for detecting the deep mineralization.

- There are important exploratory information in the low frequency classes of the S-A fractal method hence filtering these frequencies will cause faults in the interpretation of geochemical data. Therefore, in this fractal method, the low frequencies of the data should not be eliminated from the dataset in the frequency domain.

- The mineralization elements in the deep mineral deposits may have low migration to the surface and cause the weak surface geochemical anomalies with the low frequency signals. Therefore, the low frequencies in the data cannot be considered merely as background values and these frequencies may indicate the deep mineralization.

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